Colorful particles and fields in low dimensions

Color extensions of spacetime symmetry ?

(HS symmetry – Vasiliev)

- Spacetime sym: X_a
- Color sym: T_i

- New sym : $X_{ai} = X_a \otimes T_i$

$$[X_a \otimes T_i, X_b \otimes T_j]$$

$$= \frac{1}{2}\{X_a, X_b\} \otimes [T_i, T_j] + \frac{1}{2}[X_a, X_b] \otimes \{T_i, T_j\}$$

anticommutator

If both spacetime and color sym can be extended to an associative alg.

then it is OK

AdS_2 sym: $O(1,2) \simeq sl(2,\mathbb{R}) \rightsquigarrow gl(2,\mathbb{R})$

AdS_3 sym: $O(2,2) \simeq sl(2,\mathbb{R}) \oplus sl(2,\mathbb{R})$

$\rightsquigarrow gl(2,\mathbb{R}) \oplus gl(2,\mathbb{R})$

Color sym: $U(N)$

$$gl(2,\mathbb{R}) \otimes U(N) \simeq U(N,N)$$

$\left[\begin{array}{ll} U(N,N): & \text{colored } AdS_2 \text{ sym} \\ U(N,N) \times U(N,N): & \quad " \quad AdS_3 \text{ sym} \end{array}\right.$

$$(I, X_a) \otimes (I, T_i)$$

$$= \underbrace{I\otimes I}_{\substack{U(1) \\ \text{of} \\ U(N,N)}} + \underbrace{X_a \otimes I}_{\substack{\text{spacetime} \\ \Downarrow \\ \text{gravity}}} + \underbrace{I \otimes T_i}_{\substack{\text{color} \\ \Downarrow \\ \text{gauge field} \\ \text{"spin 1"}}} + \underbrace{X_a \otimes T_i}_{\substack{\text{colored spacetime sym} \\ \Downarrow \\ \text{colored "massless"} \\ \text{spin 2}}}$$

Applicat° of color sym

- CS formulat° of 3d colored gravity
  w/ Gwak, Mkrtchyan, Rey

- 3d Colorful particles

  w/ Gomis, Kleinschmidt, Mkrtchyan.

- BF formulat. of 2d colored JT gravity

  w/ Alkalaev, Yoon

World line Particle action from Symmetry

- Method : Coadjoint orbit

  (more general treatment w/ Basile, Treshan)

ex) Massive scalar ptcl in Mink d

$iso(1, d-1) = \text{Span}\{ P_\mu, J_{\mu\nu} \}$

$iso(1, d-1)^* = \text{Span}\{ \tilde{P}^\mu, \tilde{J}^{\mu\nu} \}$

$\phi \in iso(1, d-1)^*$

$\phi = p_\mu \tilde{P}^\mu + j_{\mu\nu} \tilde{J}^{\mu\nu}$

$\phi = m \tilde{P}^0 \quad (p_\mu = (m, 0, 0, 0), \; j_{\mu\nu} = 0)$

$\boxed{\text{KKS}}$

$\Rightarrow \quad S = -i \int \langle \phi, g^{-1} dg \rangle$

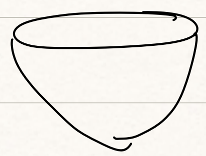
$$g = e^{i x^\mu P_\mu} e^{i v^a J_{0a}} e^{i\theta^{ab} J_{ab}}$$

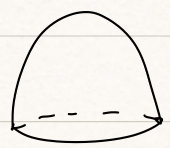
$$S = \int \langle \underbrace{e^{i v^a J_{0a}}\, m \tilde{p}^0\, e^{-i v^a J_{0a}}}_{=\, p_\mu \tilde{p}^\mu}, dx^\mu P_\mu \rangle$$

$$p_\mu p^\mu = m^2 , \quad p_0 > 0$$

$$= \int \sqrt{p_i^2 + m^2}\, dx^0 + p_i dx^i$$

$$\Longrightarrow \int p_\mu dx^\mu + e(p^2 - m^2) \qquad \text{(constrained sys)}$$





Colorful extension (flat contract. of $O(N,N) \times U(N,N)$)

$$P_\mu, M_\mu \to P_\mu, M_\mu, Q_i, N_i, P_\mu^i, M_\mu^i$$

$$\phi \in g^* , \qquad \phi = m \hat{p}^0$$

$$S = -i \int \langle \phi, g^{-1} dg \rangle = \cdots\cdots$$

constraind act.

$$S = \int \mathrm{Tr}\left[\mathbb{P}\dot{\mathbb{X}} + \mathbb{L}(\mathbb{P}^2 - m\mathbb{I})\right]$$

$\mathbb{X}, \mathbb{P}$: $SU(N,N)$ elements

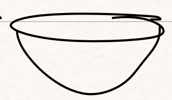
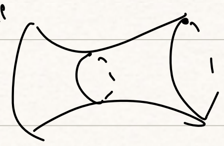
$\mathbb{L}$: $u(N,N)$ element.

$N=1$ $\quad \mathbb{X}, \mathbb{P}$: 2×2, $\qquad \mathbb{L}$: $e\,\mathbb{1}$.

$$\int \mathrm{Tr}\left[\mathbb{P}\dot{\mathbb{X}} + e(\mathbb{P}^2 - m\mathbb{I})\right]$$

$$= \int p_\mu \dot{x}^\mu + e(p^2 - m^2)$$

$N=2$. $\qquad$ 3 orbits

"  " $\qquad$ "  " $\qquad$ "  "

$$\phi = m\tilde{p}^0 = m\begin{pmatrix}1&&&\\&1&&\\&&-1&\\&&&-1\end{pmatrix} \qquad \phi = m\begin{pmatrix}1&&&\\&-1&&\\&&1&\\&&&-1\end{pmatrix} \qquad \phi = m\begin{pmatrix}-1&&&\\&-1&&\\&&1&\\&&&1\end{pmatrix}$$

In general.

$N+1$ orbits. $\qquad \underbrace{l = 0, \ldots, N}_{N+1}$

$$\phi = m \begin{pmatrix} \overbrace{\begin{matrix}1&&\\&\ddots&\\&&1\end{matrix}}^{N}\Big|\,l & & \\ & \begin{matrix}-1&&\\&\ddots&\\&&-1\end{matrix} & \\ & & \begin{matrix}1&&\\&\ddots&\\&&1\end{matrix} \\ & & & \begin{matrix}-1&&\\&\ddots&\\&&-1\end{matrix}\end{pmatrix}$$

$$\mathcal{O}_l \simeq \frac{U(N,N)}{U(\ldots)\times U(\ldots)}$$

$\mathcal{O}_f \qquad O(D-2,2)\times O(D-2,2)$

BF , Schwarzian

$$S = k\int_M \mathrm{tr}(\Phi F) - \frac{k}{2}\int_{\partial M} \mathrm{tr}(\Phi A)$$

w/ $\quad (\Phi + \gamma A_u)\big|_{\partial M} = 0$

$$\Rightarrow \quad S \simeq \frac{k\gamma}{2}\int_{\partial M} \mathrm{tr}(A_u^2)\, du$$

- Solve $F=0$
- Asymp. AdS condit (AAC)

$$A = b^{-1} a\, du\, b + b^{-1} db \qquad \left[\begin{array}{l} b = e^{rL_0} \\ a = a(u) \end{array}\right.$$

$$a = g^{-1}\dot{g}$$

$$\text{AAC} \Rightarrow \quad a = \underbrace{L_1 + \mathcal{L}(u)\, L_{-1}}_{\text{gen of } sl_2(\mathbb{R}) = \text{Span}\{L_{-1}, L_0, L_1\}}$$

$$= \begin{pmatrix} 0 & -\mathcal{L}(u) \\ \underbrace{1}_{\text{AdS B.g}} & 0 \end{pmatrix}$$

$$\int \mathrm{Tr}(A_u^2)\,du = \quad -2\int du\, \mathcal{L}(u)$$

Zero temperature

$$g = \begin{pmatrix} 1 & 0 \\ f & 1 \end{pmatrix} \begin{pmatrix} b^{-1} & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} 1 & -e \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{L}_0 = b^{-1}\ddot{b} - 2(b^{-1}\dot{b})^2 , \qquad b^2 = \dot{f}$$

$$= \frac{1}{2}\,\mathrm{Sch}(f, u)$$

Finite temperature

$$f = \tan\frac{\theta}{2}$$

$$\mathcal{L}_\beta = b^{-1}\ddot{b} - 2(b^{-1}\dot{b})^2 + b^4 , \qquad b^2 = \frac{1}{2}\dot{\theta}$$

$$= \frac{1}{2}\left(\mathrm{sch}(\theta, u) + \frac{1}{2}\dot{\theta}^2 \right)$$

Iwasawa decomp.

$$g(\theta, b, e) = \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} b^{-1} & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} 1 & -e \\ 0 & 1 \end{pmatrix}$$

$$\Rightarrow \quad \mathcal{L}_\beta$$

Color extension

$$a(u) = \underbrace{L_1 \otimes I}_{\color{red}{AdS_2\ bd.}} + \underbrace{i\, I \otimes T^A J_A}_{\mathbb{J}} + \underbrace{L_{-1} \otimes I\, \mathcal{L}(u) + L_{-1} \otimes T^A K_A(u)}_{L_{-1} \otimes \mathbb{L}}$$

$$= \begin{pmatrix} i\mathbb{J} & -\mathbb{L} \\ \mathbb{I} & i\mathbb{J} \end{pmatrix}$$

$$\mathrm{Tr}(a^2) = -2\,\mathrm{Tr}(\mathbb{L} + \mathbb{J}^2)$$

$$\mathrm{Tr}(\mathbb{L} + \mathbb{J}^2) = \boxed{\begin{aligned} &\mathrm{tr}\Big(b^{-1}\ddot{b} - 2(b^{-1}\dot{b})^2 - (d^{-1}\dot{d})^2 \\ &\qquad - \dot{d}d^{-1}(b^{-1}\dot{b} - \dot{b}b^{-1}) \Big) \\ &b^2 = \dot{f} \end{aligned}}$$